AN EFFICIENT ANALYSIS OF FLAT SLAB STRUCTURES USING SUPER-ELEMENTS

Dong-Guen Lee[1], Hyun-Su Kim[2], Hye-Sook Kim [3] and Seung-Jae Lee [4]

ABSTRACT : Flat slab system has been adopted in many buildings constructed recently because of the advantage of reduced floor heights to meet the economical and architectural demands. Structural engineers commonly use the equivalent frame method (EFM) with equivalent beams proposed by Jacob S. Grossman in practical engineering for the analysis of flat slab structures. However, in many cases, when it is difficult to use the EFM, it is necessary to use a refined finite element model for an accurate analysis. But it would take significant amount of computational time and memory if the entire building structure were subdivided into a finer mesh. An efficient analytical method is proposed in this study to obtain accurate results in significantly reduced computational time. The proposed method employs super elements developed using the matrix condensation technique and fictitious beams are used in the development of super elements to enforce the compatibility at the interfaces of super elements. The stiffness degradation of flat slab system considered in the EFM was taken into account by reducing the elastic modulus of floor slabs in this study. Static and dynamic analyses of example structures were performed and the efficiency and accuracy of the proposed method were verified by comparing the results with those of the refined finite element model and the EFM.

KEYWORDS : Flat slab structure, stiffness of slab, stiffness degradation, matrix condensation, super element, fictitious stiff beam

1. INTRODUCTION

Flat slab system in which columns directly support floor slabs without beams is adopted for many building structures recently constructed. Since flat slab system has no beams, flat slab system exhibit several advantages such as providing lower building height, good lighting, good ventilation and easier formwork which consequently make construction time shorter. However, flat slab system has some difficulties in making long span structures and large openings in slabs. Especially stiffness degradation is noticeable under lateral loads and it may be necessary to place some appropriate bearing walls in regard to structural plan because of large story drift. Structural engineers commonly use the equivalent frame method (EFM) in practical engineering for the analysis of flat slab structures. Determining the effective width of a slab is most important in the EFM and many researches on an effective width have been performed(Pecknold 1975, Allen 1977). Above all, the calculation method of effective width(Grossman 1997) proposed by Jacob S. Grossman is widely used in practical engineering. Grossman modified existing methodology to describe the degradation of the stiffness of the slabs due to the level of the lateral loads by using the stiffness degradation factor based on the tests(Moehle 1990) performed at U.C. Berkeley. In this study, some restrictions of the EFM widely used in practical analysis are presented and an efficient analysis method that can supplement those defects is proposed. Floor slabs are subdivided into many finite elements to analyze the slabs having irregular shape or openings in this study. An efficient analysis method that could provide accurate results was proposed in this study to reduce computational time drastically using super elements derived by introducing fictitious beams(Lee 2002, Kim 2003). It was said in Grossman's study that the stiffness degradation of the slabs due to the lateral loads could not be included in the general finite element method(FEM). However, the analysis method that can consider the stiffness degradation according to lateral drifts by adjusting elastic modulus of the slabs was proposed in this study based

[1] *Professor, Department of Architectural Engineering, Sungkyunkwan University, Korea.*
[2] *Research Associate, Department of Architectural Engineering, Sungkyunkwan University, Korea.*
[3] *Graduate Student, Department of Architectural Engineering, Sungkyunkwan University, Korea.*
[4] *Graduate Student, Department of Architectural Engineering, Sungkyunkwan University, Korea.*

on the stiffness degradation factor proposed by Grossman. Static and dynamic analyses of example structures having various types of plans were performed to verify the efficiency and accuracy of the proposed method.

2. MODELING FLAT SLAB USING EQUIVALENT FRAME METHOD

2.1 Grossman's methodology for effective width

Various studies on resistance capacity for lateral loads were performed by previous researchers. And new methodology for effective width was proposed by Grossman as shown in **Equation (1)** through modifying and improving existing methodologies of the EFM.

$$\alpha l_2 = K_d[0.3l_1 + C_1(l_2/l_1) + (C_2 - C_1)/2](d/0.9h)(K_{FP}) \quad (1)$$

with limits: $(0.2)(K_d)(K_{FP})l_2 \le \alpha l_2 \le (0.5)(K_d)(K_{FP})l_2$

where, α = equivalent width factor

αl_2 = effective width of slab at center line of support

K_d = factor considering degradation of stiffness of slabs at various lateral load levels

l_1 = length of span of supports in direction parallel to lateral load

l_2 = length of span of supports in direction transverse to lateral load

C_1 = size of support in direction parallel to lateral load

C_2 = size of support in direction transverse to lateral load

d = effective depth of slab h = slab thickness

K_{FP} = factor adjusting αl_2 at edge exterior and corner supports

(1.0 for interior supports, 0.8 for exterior and edge supports, 0.6 for corner supports)

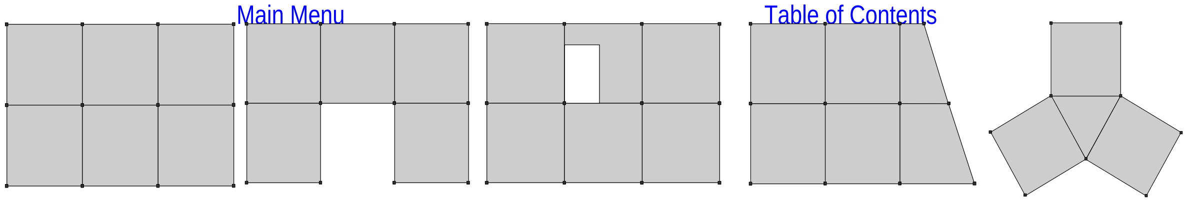
In the case of exterior column, adjustments are made by multiplying effective width(αl_2) by **Equation (1)** by $[l_3 + (l_2/2)]/l_2$ where l_3 equals the distance between the column centerline and the parallel edge of the slab. K_d in **Equation (1)** is the stiffness degradation factor and the values in **Table 1** are used for K_d according to lateral drift.

Table 1. Stiffness Degradation Factor according to Lateral Drift

Lateral drift	$h_s/800$	$h_s/400$	$h_s/200$	$h_s/100$
K_D	1.1	1.0	0.8	0.5

2.2 Problems of equivalent frame method

The EFM frequently used in practical engineering for the analysis of flat slab structure has some limitations in application range and accuracy because slabs are modeled by equivalent beams based on buildings with regular arrangements of columns. The EFM can be easily applicable to the flat slab structure having rectangular plan shown in **Figure 1(a)**. However, the EFM cannot be applied to the flat slab structures having irregular plans shown in **Figure 1(b), 1(c), 1(d)** and **1(e)**. In the case of plan shown in **Figure 1(b)**, three quadrants around the middle column have slab but one quadrant has no slab. The factor concerning column location(K_{FP}) for that case was not used in the Grossman's methodology. It is difficult that the EFM is applied to the structures having slab with openings shown in **Figure 1(c)**, but commercial buildings using flat slab system usually have slabs with openings for escalators or equipments. Thus application of the EFM is limited. Since length of span(l_1, l_2) and column location factor(K_{FP}) cannot be easily determined about the structures having plans shown in **Figure 1(d)** and **1(e)**, the application of the EFM to those structures is impossible. The EFM cannot accurately represent the stress distribution that is most important results for the design of slabs, therefore flat slabs are designed using the shear forces and the moments of equivalent beams. Although the EFM has various problems mentioned above, it is used by most of the engineers in practice because there is no appropriate analysis method for the flat slab structure.



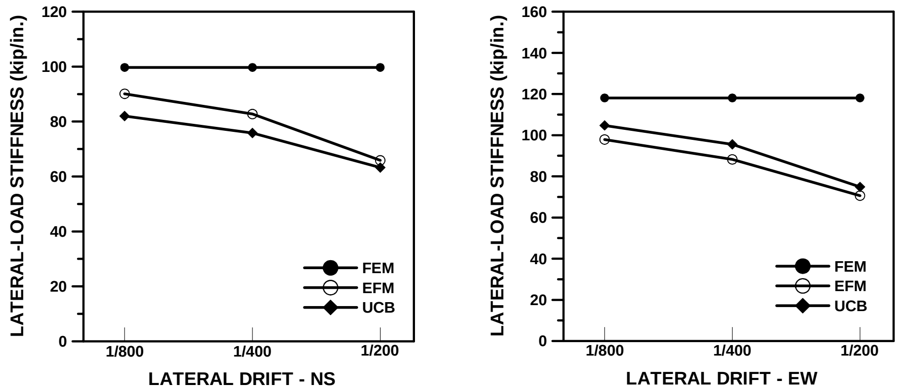
(a) (b) (c) (d) (e)

Figure 1. Various types of plans of flat slab structure

3. ANALYSIS OF FLAT SLAB STRUCTURE USING FINITE ELEMENT METHOD

3.1 The stiffness degradation of slabs according to lateral drift

The stiffness degradation of slabs is usually remarkable in the case of flat slab structures under lateral loads. Therefore, Grossman proposed the stiffness degradation factor(K_D) that can reduce the effective width according to lateral drift in his study based on the tests performed by Prof. Moehle at U.C. Berkeley (Moehle 1990). **Figure 2** shows the variation of lateral stiffness according to lateral drift in EW and NS direction regarding the test structure at U.C. Berkeley(UCB). It can be noticed that the lateral stiffness of the structure predicted by the EFM is constant regardless of lateral drift but that of UCB is reduced by the increment of lateral drift. And the EFM can represent the reduction of the lateral stiffness of structure because the methodology proposed by Grossman(EFM) reduce effective width using the stiffness degradation factor according to lateral drift. However, graphs shwon in **Figure 2** represent the stiffness degradation of whole structure. The stiffness degradation factor of only slabs is computed in this study to compare with Grossman's sutdies as shown in **Figure 3**. The analysis results of the FEM using reduced elastic modulus of slabs by stiffness degradation fator in **Figure 3** are very similar to those of the EFM as shown in **Figure 4**. Thus, the FEM using reduced elastic modulus by stiffness degradation fator can consider the stiffness degradation of slabs under lateral loads.



(a) (b)

Figure 2. Variation of lateral stiffness of flat slab structure according to lateral drift

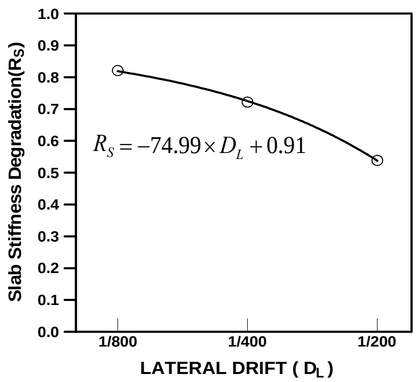
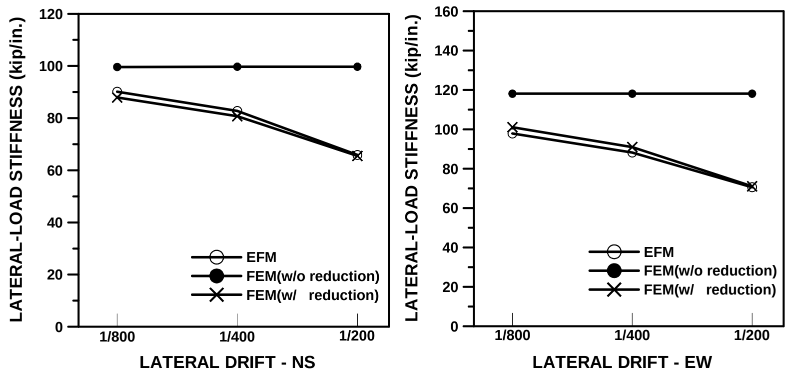


Figure 3. Stiffness degradation factor of slab

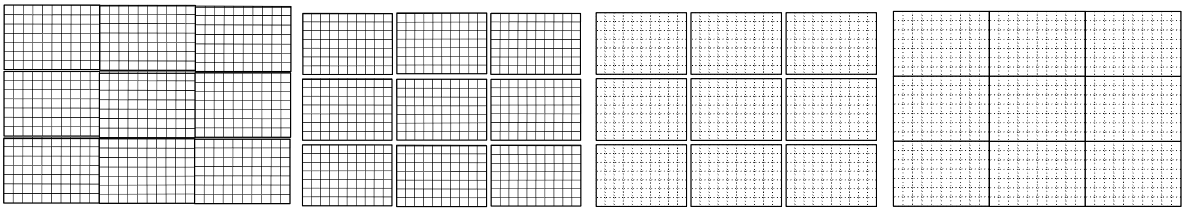


(a) (b)

Figure 4. Results of FEM with stiffness reduction

4. DEVELOPMENT OF SUPER ELEMENTS FOR FLAT SLAB

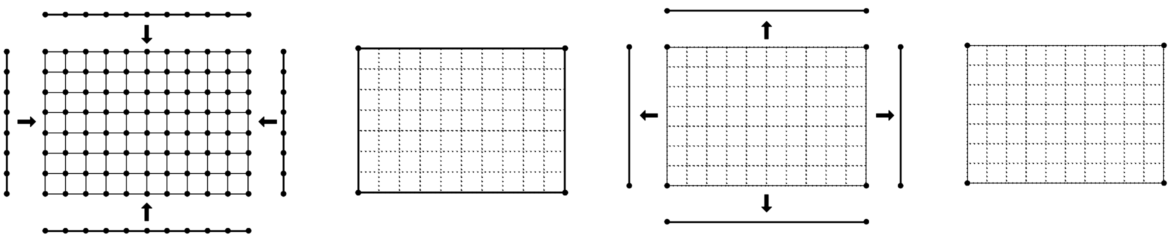
Most of the flat slab structures have repeated arrangement of columns with the same architectural plan on each floor. Modeling procedure of flat slab system using super elements is shown in **Figure 5**. **Figure 5(a)** shows refined mesh model for the typical flat slab system using many finite elements. This refined mesh model can be separated into substructures having the same configuration as shown in **Figure 5(b)**. If all of the DOF's except those of the node for columns are eliminated by using the matrix condensation technique, super elements illustrated in **Figure 5(c)** could be generated. Then the flat slab is constructed by joining the left nodes of a generated super elements as shown in **Figure 5(d)**.



(a) (b) (c) (d)

Figure 5. Modeling procedure of flat slab system using super elements

Fictitious stiff beams were introduced to enforce the compatibility conditions at boundaries of super elements in this study. **Figure 6** illustrates how to use fictitious beams in the process of developing super elements for flat slabs. Fictitious beams were added to the interfaces of a substructure as illustrated in **Figure 6(a)**. Then, all of the DOF's in the substructure except the DOF`s at the corners for columns was eliminated by using the matrix condensation technique as shown in **Figure 6(b)**. Surplus stiffness introduced by fictitious beams was eliminated by subtracting the flexural stiffness of fictitious beams without internal nodes from the stiffness matrix of the super element as shown in **Figure 6(c)**. As a result, the stiffness matrix of super element can be obtained in **Figure 6(d)**.



(a) (b) (c) (d)

Figure 6. Use of fictitious beams for flat slab system

5. ANALYSIS OF EXAMPLE STRUCTURES

Static and dynamic analyses of flat slab struructures as shown in **Figure 7** were performed to verify the accuracy and the efficiency of the proposed analysis method. The bending stiffness of slab was reduced in the FEM and the proposed method by modifying elastic modulus. As shown in **Figure 8** and **9**, all of the methods provide an almost identical results in the lateral displacements and natural periods. Time history analysis was performed to verify the accuracy in prediction of seismic response of building structures. The ground acceleration time history of the El Centro earthquake (NS 1940) was used as the input ground motion as shown in **Figure 10**. The results of time history analysis indicate that the analysis results of all methods almost identical like static analysis. The computational time and the number of DOF`s used for the analyses are compared in **Table 2**. The proposed analysis method could provide accurate result with only 0.1% of the computational time compared to the refined mesh model. Although the computational time of the EFM is very short compared to that of FEM, the EFM cannot represent the stress distribution of slabs. But the proposed can accurately represent the stress distribution. The flat slab structure shown in **Figure 12** was used for the investagation of the flat slab having openings. Since the EFM cannot be applied to the flat slab structure with openings, the EFM was excluded in the analysis of that structure. The lateral displacements and natural periods of the proposed method were almost identical to those of the FEM as shown in **Figure 13** and **14**.

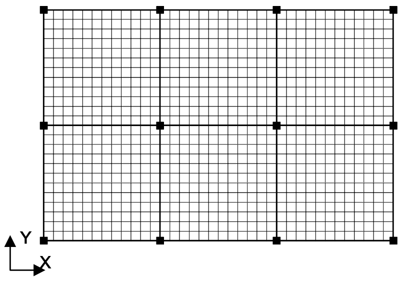


Figure 7. Plan of example structure

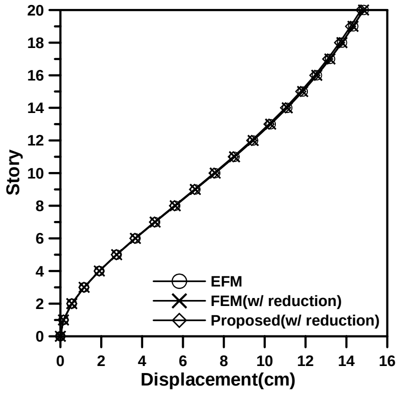


Figure 8. Lateral displacements

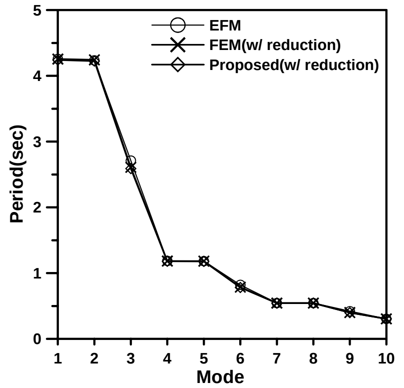


Figure 9. Natural periods of vibration

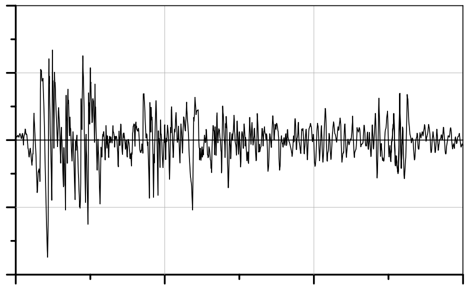


Figure 10. El Centro(NS, 1940) seismic load

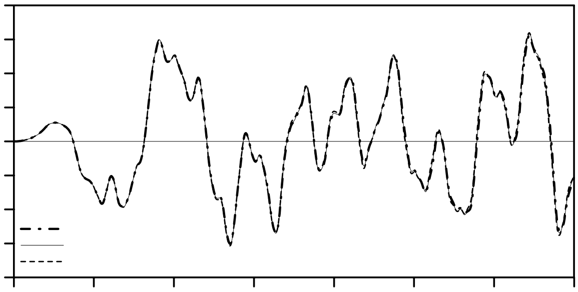


Figure 11. Roof displacement time history

Table.2 Number of DOF's and computational time for analysis

Models	Number of DOF's	Computational time(sec)				
		Assembly M&K	Static analysis	Eigenvalue analysis	Time history analysis	Total
EFM	1740	2.61	0.36	19.69	7.67	30.33
Proposed	780	13.70	0.12	5.75	3.36	22.93
FEM	55500	230.22	394.38	17406.66	281.58	18312.84

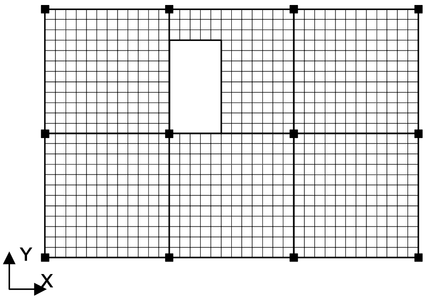


Figure 12. Plan of example structure having opening

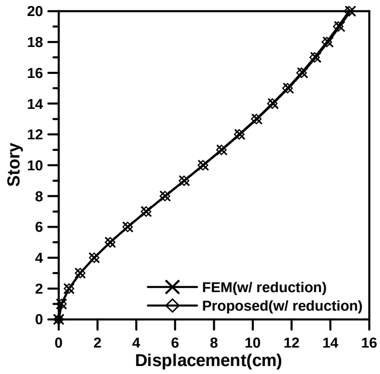


Figure 13. Lateral displacement

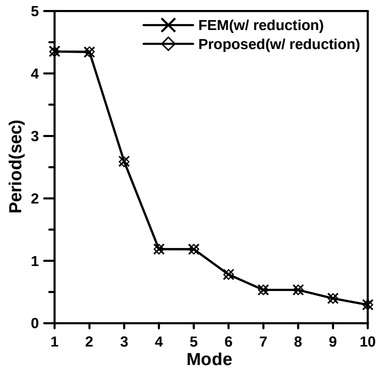


Figure 14. Natural period of vibration

6. CONCLUSIONS

An efficient analysis method was proposed in this study using super elements and fictitious beams for the analysis of flat slab strudture. The accuracy and the efficiency of the proposed method were investigated by performing analysis of various example structures. The major observations and findings are summarized as follows:

(1) The behavior of flat slab structure having typical rectangular plan can be well predicted by equivalent frame method. But in the case of structures having irregular shape or slabs with openings, equivalent frame method cannot be used.
(2) The stiffness degradation of flat slab system was taken into account by reducing the elastic modulus of floor slabs in the FEM. The stiffness degradation factor of slabs was derived from the stiffness degradation factor used in Grossman's study. The static and dynamic analysis of flat slab structure can be performed more accurately by using derived stiffness degradation factor.
(3) If the finite element method considering the stiffness degradation of slabs is used, the structural analysis of flat slab structure having irregular shape or slabs with openings can be performed and stress distribution of floor slabs can be easily represented.
(4) The analysis results of the proposed method were very similar to those of the refined mesh model in static and dynamic analysis. Computational time and computer memory for analyses of the proposed method could be drastically reduced.

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